HEAT EXCHANGE ON THE INITIAL THERMAL SECTION IN STABILIZED TURBULENT AIR FLOW IN CIRCULAR PIPES AND RECTANGULAR CHANNELS

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Formulas are proposed for an analysis of the local and mean heat exchange on the initial thermal section for stabilized turbulent air flow in circular pipes and rectangular channels on the basis of an extension of test results.

The regularities of the heat exchange process on the initial thermal section in pipes for a turbulent flow mode of the air stream and a completely developed profile of the velocity field have been investigated in detail both theoretically and experimentally [5, 8, 10, 11]. In the standard formulation the problem is reduced to determining the dependence $\alpha_{X_1} = \varepsilon_{X_1} \alpha_\infty$ in which the correction "in the initial section" ε_{X_1} is a function of the Reynolds number and a dimensionless coordinate measured from the beginning of the heating $\varepsilon_{X_1} = f(\mathrm{Re}_d;\ X_1/d)$. A comparison between numerical solutions and test data of various authors in [6] shows that the stabilization of local heat exchange is performed within a length equal to fifteen to twenty equivalent diameters.* The following singularities are hence observed in the configurations of the curves $\varepsilon_{X_1} = f(\mathrm{Re}_d;\ X_1/d)$. In the domain of the initial thermal section investigated best, where $X_1/d = 1.5$-20, the corrections ε_{X_1} are lowered smoothly from 1.2-1.4 to one and depend slightly on the Reynolds number. The results of the majority of papers can here be generalized, with a ± 10% error, as a function of just X_1/d. Relationships of the type mentioned have been proposed in [5, 6], say. For $X_1/d < 1.5$ the corrections ε_{X_1} grow rapidly to the values 2-3 in direct proximity to the beginning of the heating. The role of the Reynolds number has also been magnified substantially. The quantity of experimental results in this area is restricted, and analytical solutions diverge from experiment and between themselves by 30-50% and more. It should be noted that the published material on the heat exchange in the initial thermal section refer primarily to circular pipes. Only the theoretical solution [8], wherein a plane-parallel channel is examined, is the exception.

Heat exchange in the initial thermal sections of four rectangular channels is studied herein for small X/d. The geometric characteristics of the channels, presented in Table 1, were chosen in such a way as to include the greatest possible interval in the parameter A/B. The tests were performed in a wide tunnel with experimental section placed at the intake of the flow-through part [3]. The test stand assures a W = 1-60 m/sec stream velocity at an air temperature of $T_0 = 290$-293°K.

* Strictly speaking, the corrections ε_{X_1} become one at infinity, however, for $X_1/d = 20$ the discrepancy between α_{X_1} and α_∞ does not even exceed 2.5%.

TABLE 1. Geometrical Characteristics of the Rectangular Channels

Parameters	Notation	Number of the channel			
		1	2	3	4
Equivalent diameter, m	$d\cdot 10^3$	40	33,3	13,8	3,84
Ratio between the sides	$\frac{A\cdot 10^3}{B\cdot 10^3}$	$\frac{40}{40}=1$	$\frac{100}{20}=5$	$\frac{100}{7,4}=13,5$	$\frac{50}{2}=25$

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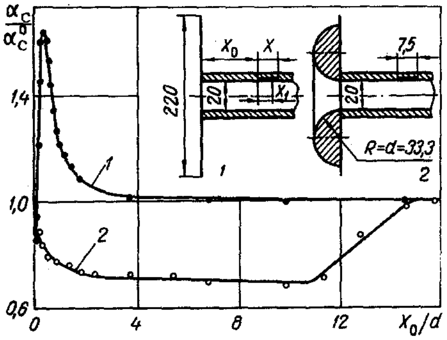

Fig. 1

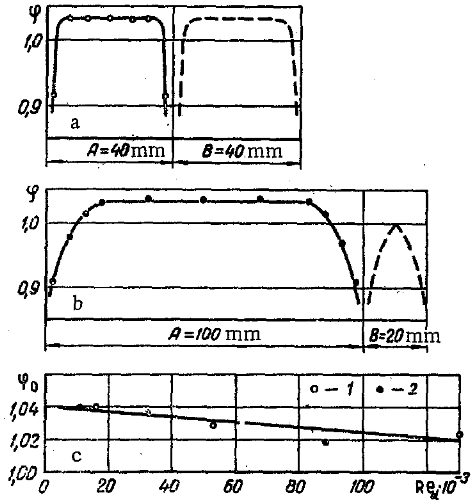

Fig. 2

Fig. 1. Distribution of the heat exchange intensity along the middle line of the wide wall of channel No. 2 (calorimeter No. 4, Re_d = 55,000: 1) inlet with a sharp edge with a 90° angle; 2) inlet edge rounded off on the arc of a circle

Fig. 2. Distribution of the heat exchange intensity ($\varphi = \alpha_c/\alpha'_x$) over the channel perimeter. a) Channel No. 1, Re_d = 50,000; b) Channel No. 2, Re_d = 50,000; c) the dependence $\varphi_0 = f(Re_d)$ [1) channel No. 1; 2) channel No. 2].

The heat exchange coefficients at different distances from the inlet and at different points of the perimeter were measured by a nonstationary method on one of the channel walls [2]. Exactly as in [2], plates of CT35 steel with imbedded Chromel–Alumel thermocouples with 0.1 mm diameter leads were used as alpha calorimeters. The geometric dimensions of the alpha calorimeters are presented in Table 2. The magnitudes of the heat exchange coefficients α_c determined in the experiments are mean values in X for T_w = const.

The inlet profile on the channels was varied in the tests. It is shown in Fig. 1 how the heat exchange coefficient α_c changes along the middle line of the wide wall of channel No. 2 for a sharp inlet edge with a 90° angle and an inlet edge rounded off along the arc of a circle as a function of the extent of the adiabatic section lying ahead of the calorimeter. The sharp edge causes separation and rapid turbulization of the boundary layer. The heat exchange intensity is stabilized at $X_0/d \approx 10$ by gradually being reduced after the separation maximum.

In the second case the curve α_c/α_c^0 indicates a mixed flow, and stabilization sets in only for $X_0/d \approx 15$. Duplication of the measurement results at some distance from the inlet is a criterion of a hydrodynamically stabilized flow. Later, heat exchange results obtained in precisely this domain will be examined. In all

TABLE 2. Geometric Characteristics of the Alpha-Calorimeters

Parameters	Notation	Number of the calorimeter					
		1	2	3	4	5	6
Dimensions of the active calorimeter surface, m	$X \cdot 10^3$	2,0	2,34	5,2	7,5	14,7	19,2
	$Z \cdot 10^3$	5,1	7,9	5,1	15,0	14,9	19,8
Reduced calorimeter length*							
Channel No. 1				0,13			
Channel No. 2	$\frac{X}{d}$	0,06		0,156	0,225	0,442	0,575
Channel No. 3			0,17		0,544		
Channel No. 4					1,95		

*Only the combinations of X/d are indicated which were used in the tests.

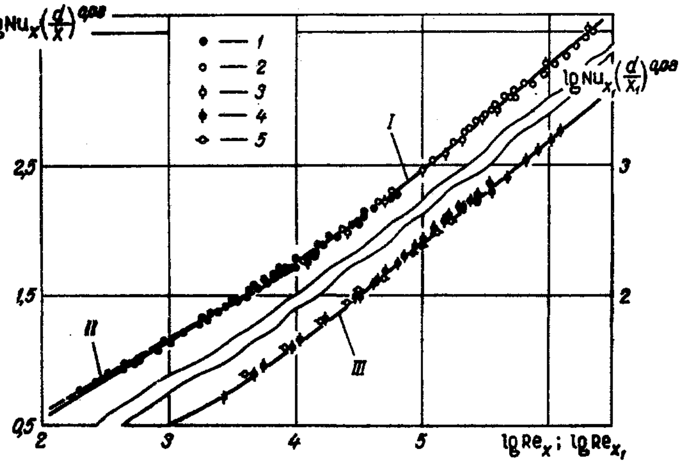

Fig. 3. Generalization of experimental data for mean and local heat transfer at initial thermal section: I) by equation (2); II) by (9); III) by (10); 1) authors' data; 2, 3) data for mean heat transfer [5, 10]; 4, 5) data for mean heat transfer [5, 10].

cases the length of the unheated section located ahead of the calorimeter would not be less than 20d. The reduced length of the heated section varied within the limits X/d = 0.06-1.95 (Table 2).

Presented in Figs. 2a and b are typical distributions of the heat exchange coefficients in the transverse direction in the hydrodynamic stabilization domain for channels with a 1:1 and 5:1 ratio between the sides. The dashed lines for the adjoining sides have been constructed under the assumption that the heat exchange in the corner is symmetric to its vertex. Such an approach is verified sufficiently well in [7] where the velocity fields have been computed for a number of rectangular channels. The main part of the measurements was performed with the calorimeters placed on the axial line of the wide wall of the channels. The mean heat exchange coefficients with respect to the perimeter have been obtained by using corrections for the nonuniformity of the heat exchange over the perimeter φ_0. As is seen from Fig. 2c, the quantity φ_0 is close to one and practically independent of A/B.

The experimental heat exchange coefficients refer to the temperature head ΔT_0. It can be shown that the temperature head ΔT_0 in a channel with constant wall temperature along the length and perimeter is related to the mean logarithmic temperature head by means of the expression

$$\frac{\Delta T_l}{\Delta T_0} = \frac{4\mathrm{St}\,\dfrac{X}{d}}{\ln \dfrac{1}{1 - 4\mathrm{St}\,\dfrac{X}{d}}}. \tag{1}$$

Because of the smallness of the quantities X/d the ratios $\Delta T_l/\Delta T_0$ are 0.995-0.975 under the experiment conditions.

Taking account of the correction φ_0 and the correction for the mean logarithmic temperature head, the heat exchange coefficients α_c^0 were processed in the criterial dependence $Nu_X(d/X)^{0.08} = f(Re_X)$, where the calorimeter dimension along the stream flow X was taken as governing. In a logarithmic anamorphosis, the dependence $Nu_X(d/X)^{0.08} = f(Re_X)$ unites all test points with a ± 10% spread, as well as the data on the

TABLE 3. Correction Factor ε_X to Compute the Mean Heat Exchange

Re_d	X/d												
	0,05	0,1	0,5	1,0	2,0	5,0	10	15	20	30	40	50	60
10000	5,10	4,10	2,48	2,02	1,67	1,39	1,26	1,19	1,15	1,09	1,05	1,02	1,00
50000	3,73	3,01	1,92	1,69	1,53	1,36	1,25	1,19	1,14	1,09	1,05	1,02	1,00
100000	3,26	2,65	1,83	1,66	1,51	1,34	1,24	1,18	1,13	1,08	1,05	1,02	1,00

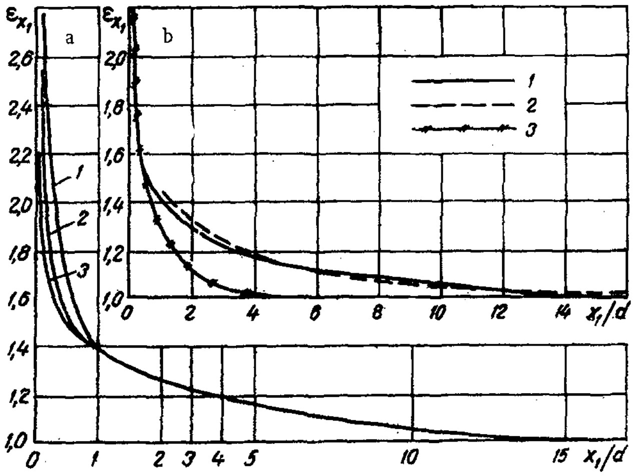

Fig. 4. The dependence $\varepsilon_{X_1} = f(X_1/d;\ Re_d)$ for the local heat exchange in the initial section: a) computation by means of (12): 1) $Re_d = 10{,}000$; 2) 50,000; 3) 100,000; b) comparison with the results of analytic solutions: 1) by means of (12), $Re_d = 50{,}000$; 2) Sparrow [11] solution, $Re_d = 50{,}000$; 3) Deissler solution [8], $Re_d = 30{,}000$.

mean heat exchange for circular pipes taken from [5] and [10]* (see Fig. 3, curve 1). The steepness of the generalizing curve increases together with the Reynolds number. For small Re_X the deviation is about 0.6, and later, for Re_X 30,000 it approaches 0.8 asymptotically. The equation approximating the curve I in the $Re_X = 2 \cdot 10^2 - 4 \cdot 10^6$ range appears to be the following:

$$Nu_X = 0.08 Re_X^{0.7} \left(\frac{X}{d}\right)^{0.08} \cdot 10^{0.1 \sqrt{(\lg Re_X - 4.4)^2 + 0.15}} . \tag{2}$$

By approximating the curve I by parts, simple formulas can be obtained:

for $Re_X < 25{,}000$

$$Nu_X = 0.22 Re_X^{0.6} \left(\frac{X}{d}\right)^{0.08} ; \tag{3}$$

for $Re_X > 25{,}000$

$$Nu_X = 0.029 Re_X^{0.8} \left(\frac{X}{d}\right)^{0.08} . \tag{4}$$

Tests have verified (2) within the limits $X/d = 0.06$–60.0, $Re_d = 3 \cdot 10^3$–$100 \cdot 10^3$, and $Pr = 0.707$ and it limits the extent of the initial thermal section to $X/d < 60$. If $X/d > 60$, the computation must be made by the equation for the stabilized heat exchange in circular pipes and channels of noncircular cross section [4]

$$Nu_\infty = 0.018 Re_d^{0.8} . \tag{5}$$

The dependence (5) is verified in [4] by experimental results for rectangular channels in the range $A/B = 1$–17.8, $Re_d > 10^4$.

Equations (2) and (5) permit a correction formula to be obtained for the computation of the mean heat exchange

* Test data on the mean heat exchange are presented in [10] in tabular form as a function of X/d. They were processed at $Re_d = 50{,}000$, which corresponds to the middle of the range investigated in [10].

$$\varepsilon_X = 4.45\mathrm{Re}_d^{-0.1}\left(\frac{X}{d}\right)^{-0.22}\cdot 10^{0.1\sqrt{\left(\lg \mathrm{Re}_d \frac{X}{d}-4.4\right)^2+0.15}}. \tag{6}$$

Values of the correction ε_X in the function X/d and the Reynolds number needed for engineering calculations are given in Table 3.

Since (5) is applicable only for the developed turbulent flow domain, then (6) becomes meaningless for $\mathrm{Re_d} < 10^4$. The mean heat exchange in the $\mathrm{Re_d} = 3\cdot 10^3$-$10^4$ band should be computed directly by means of (2).

A theoretical analysis of the mean heat exchange on short surfaces in X with a constant wall temperature has been performed in [9]. The case is considered when the thermal boundary layer is completely within the laminar sublayer. As has been shown in [9], the heat exchange intensity is hence determined by the magnitude of the tangential stresses at the wall

$$\mathrm{Nu}_X = 0.807\left(\frac{X^2}{\mu a}\right)^{\frac{1}{3}}\tau_w^{\frac{1}{3}}. \tag{7}$$

The mean tangential stress over the perimeter is calculated by the Blasius law for stabilized turbulent flow in pipes and rectangular channels:

$$\frac{8\tau_w}{\rho w^2} = \frac{0.316}{\mathrm{Re}_d^{0.25}}. \tag{8}$$

After substituting (8) and (7) and converting to the governing dimension X for the Prandtl number 0.707, we obtain

$$\mathrm{Nu}_X = 0.245\mathrm{Re}_X^{0.585}\left(\frac{X}{d}\right)^{0.0835}. \tag{9}$$

The heat exchange coefficient in (7) and (9) is referred to the temperature head ΔT_0. It has been remarked above that for small X the ratio $\Delta T_l/\Delta T_0$ is almost one, and this means that a direct comparison between (9) and (3) turns out to be possible. The curve II computed by means of (9) agrees with curve I to 5% accuracy in Fig. 3.

It is pertinent to turn attention to the fact that the measurements in [5] correspond to a constant wall temperature, and in [10] to a constant heat flux. Judging by the results of a generalization of the mean heat exchange coefficients, the boundary conditions exert no noticeable influence on the heat exchange of the initial thermal section.

A subsequent analysis of the local heat exchange on the initial thermal section has been performed by differentiating the expression for the mean heat exchange coefficient resulting from (2). After differentiation and reduction of the formula to criterial form, we obtain

$$\mathrm{Nu}_{X_1} = 0.08\mathrm{Re}_{X_1}^{0.7}\left(\frac{X_1}{d}\right)^{0.08}\cdot 10^{0.1\sqrt{(\lg \mathrm{Re}_{X_1}-4.4)^2+0.15}}\left[0.78 + \frac{0.1\,(\lg \mathrm{Re}_{X_1}-4.4)}{\sqrt{(\lg \mathrm{Re}_{X_1}-4.4)^2+0.15}}\right]. \tag{10}$$

Curve III in Fig. 3 corresponds to (10). Superposed there are experimental results on the local heat exchange in circular pipes taken from [5, 10]. The spread of the experimental points relative to curve III does not exceed ± 10% . Therefore, the selection of the equivalent diameter as parameter characterizing the cross section permits extension of the test data on the mean and local heat exchange in circular pipes and rectangular channels. In order to simplify the computational operations, curve III in Fig. 3 has been approximated by an equation similar in structure to the equation [2]:

$$\mathrm{Nu}_{X_1} = 0.062\mathrm{Re}_{X_1}^{0.7}\left(\frac{X_1}{d}\right)^{0.08}\cdot 10^{0.1\sqrt{(\lg \mathrm{Re}_{X_1}-3.9)^2+0.01}}. \tag{11}$$

Formulas (10) and (11) agree to 2% accuracy within the interval $\mathrm{Re}_{X_1} = 2\cdot 10^2$-$2\cdot 10^6$.

By using (11) and (5), we can arrive, in the long run, at an expression for the correction ε_{X_1}

$$\varepsilon_{X_1} = 3.45\mathrm{Re}_d^{-0.1}\left(\frac{X_1}{d}\right)^{-0.22}\cdot 10^{0.1\sqrt{\left(\lg \mathrm{Re}_d \frac{X_1}{d} - 3.9\right)^2 + 0.01}}. \tag{12}$$

Graphs constructed by means of (12) are presented in Fig. 4a, which show that stabilization of the local heat exchange occurs at the length $X_1/d = 16.5$. For $\mathrm{Re}_d X_1/d > 10^4$ the correction ε_{X_1} depends only on X_1/d and agrees well with the computation using a formula proposed in [11]

$$\varepsilon_{X_1} = 1.38\left(\frac{X_1}{d}\right)^{-0.12}. \tag{13}$$

A comparison with the results of theoretical solutions for circular pipes in Fig. 4b shows that (12) actually confirms the results in [11] for $X_1/d > 0.5$ and agrees well with the solution in [8] for $X_1/d < 0.5$.

NOTATION

A	channel width, m;
B	channel height, m;
d	equivalent channel diameter, m;
X_0	extent of the initial adiabatic section, m;
X	total heated length, m;
X_1	running coordinate along the heated length;
Z	cross-flow dimension of the calorimeter, m;
w	mean flow rate, m/sec;
T_0	air temperature at the inlet, °K;
T_W	wall temperature, °K;
ΔT_0	temperature head, calculated by means of the air temperature at the inlet, °K;
ΔT_l	mean logarithmic temperature head, °K;
α_c	coefficient of calorimeter heat exchange, W/m² ·deg;
α_c^0	heat exchange coefficient of a calorimeter located on the axial line of the wall in the stabilized flow zone, W/m² ·deg;
α_X	mean heat exchange coefficient over the length and the perimeter of the channel, W/m² ·deg;
α'_X	mean heat exchange coefficient over the length and the perimeter referred to ΔT_0, W/m² ·deg;
α_{X_1}	local heat exchange coefficient averaged over the channel perimeter, W /m² ·deg;
α_∞	heat exchange coefficient in the stabilized heat exchange domain, W/m² ·deg;
$\mathrm{Re}_d = wd/\nu$	Reynolds number computed with respect to the equivalent channel diameter;
$\mathrm{Re}_X = wX/\nu$; $\mathrm{Re}_{X_1} = wX_1/\nu$	Reynolds number computed with respect to the lengths X and X_1;
$\mathrm{Nu}_X = \alpha_X X/\lambda$, $\mathrm{Nu}_{X_1} = \alpha_{X_1}X_1/\lambda$	mean and local Nusselt numbers;
$\mathrm{Nu}_\infty = \alpha_\infty d/\lambda$	Nusselt number computed for the stabilized heat exchange domain;
$\varepsilon_X = \alpha'_X/\alpha_\infty$, $\varepsilon_{X_1} = \alpha_{X_1}/\alpha_\infty$	correction factors to compute the mean and local heat exchange;
$\mathrm{St} = \alpha_{X_1}/w\rho c$	Stanton number;
τ_W	tangential stress on the wall, N/m²;
$\varphi_0 = \alpha_c^0/\alpha'_X$	ratio between the heat exchange coefficient measured on the axial line of the wall and the mean value over the perimeter.

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